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Blood-Gas Transfer in an Axial Flow Annular Exchanger

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The rates of hemoglobin saturation and carbon dioxide reduction in blood flowing in an annular-shaped membrane exchanger were studied theoretically by considering two limiting cases: the fluid-limited case and the wall-limited case. The blood is assumed to flow axially with a fully developed laminar profile. In the fluid-limited analysis the membrane wall is assumed to be infinitely permeable, so the efficiency of the oxygenator depends solely on the rate of gas transport within the blood. In the wall-limited case the blood is assumed to have infinite transport ability, thus the efficiency of the oxygenator is limited by the diffusion rate through the wall. For realistic radii ratio, the oxygenator is fluid limited, thus suggesting the need for enhanced blood mixing. In the fluid-limited case the rate of gas transport depends on the radii ratio and a dimensionless length, which is independent of the cylinder sizes. If $P_{O_2} = 715$ mm. Hg and $P_{CO_2} = 0$ mm. Hg are maintained at both walls, oxygenation is the slower process. For the same flow rate the length required for complete hemoglobin saturation in the annulus with an inner to outer radius ratio of 0.95 is 1/65 of that required in a circular tube.

Auxiliary blood-gas exchange devices, commonly (although incorrectly) known as oxygenators, are generally of three types: bubble, film, and membrane. Of these three types the membrane oxygenator is the least developed, but considered by many to be most promising (*1*). A membrane oxygenator consists of a closed, semipermeable conduit submerged in an oxygen-rich atmosphere. The blood flows in the conduit and receives oxygen and expels carbon dioxide through the conduit wall. Recently, several investigators (*2* to *6*) have studied the properties of membrane oxygenators with both experiments and mathematical analyses and have, in general, been successful in their theoretical predictions. The present paper utilizes a mathematical technique, which has been experimentally substantiated for other geometries, for the study of gas transfer in an axial flow annular exchanger. This paper, together with a subsequent paper on channel flow exchangers, is intended to supplement the previous papers on circular tube flow and provide overall guidelines for designing oxygenators.

Since the resistance to mass transfer comes from two sources, the membrane and the blood, some knowledge of the overall rate of gas exchange may be gained by considering two limiting cases. The first limiting case, referred to as the fluid-limited case, assumes that the wall flux so exceeds the flux in the blood that the wall may be neglected. The second limiting case, referred to as the wall-limited case, assumes that the lateral gas transfer in the blood phase is infinitely fast and the efficiency of the oxygenator is limited by the diffusion rate through the membrane.

In the fluid-limited analysis the gas transport process is described by two convective diffusion problems, one for oxygen and one for carbon dioxide, with generation or loss of material by instantaneous chemical reactions. Due to the Bohr and Haldane effects, these two problems are not independent of each other. The mutual dependency manifests itself at the local level. In large-scale systems, such as these auxiliary lungs, the interdependent effect is small and it is practical to treat the two diffusion processes separately, with some provisions for the Bohr and Haldane effects on each of them.

FLUID-LIMITED ANALYSIS

The analysis of the fluid-limited case for the blood gas

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transport is based on solving the convective-diffusion equation

$$[1 + f(C)]\,\mathbf{V} \cdot \nabla C = D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + \frac{\partial^2 C}{\partial Z^2}\right] \tag{1}$$

The term $f(C)$, representing the sink characteristics of oxygen or carbon dioxide, can be derived from the appropriate dissociation curve (2, 3). For oxygenation of cattle blood, used in experiments

$$f(C) = 234 \exp(-2.57 \times 10^4 C) + 40.6 \exp(-2.25 \times 10^4 C + 1.14) \tag{2}$$

and for carbon dioxide removal

$$f(C) = 9.0 \tag{3}$$

The $f(C)$ for oxygenation is an accurate curve fit; the $f(C)$ for carbon dioxide is only a linearization of the absorption curve, and as such is a first-order approximation.

Since the shear rates in such devices are usually large, the blood can be assumed to be a Newtonian fluid, and if the flow in an annulus is laminar and axial, the velocity field $\mathbf{V}$ is well known. The axial component is

$$V_Z = \frac{r_o^2}{4\mu}\frac{\partial p}{\partial Z}\left[1 - (r/r_o)^2 + \frac{(r_i/r_o)^2 - 1}{\ln(r_i/r_o)}\ln(r/r_o)\right] \tag{4}$$

For the oxygenation problem, Equation (1) can be simplified by using the following dimensionless variables:

$$r^* = r/r_o, \tag{5}$$

$$r_i^* = r_i/r_o, \tag{6}$$

$$Z^* = \frac{Z}{(r_o + r_i)\,N_{Pe}} = \left(\frac{\pi D}{2Q}\right) Z, \tag{7}$$

$$V_Z^* = \frac{V_Z}{2V_m} = \frac{1}{2V_m}\frac{r_o^2}{4\mu}\frac{\partial p}{\partial Z}\left[1 - r^{*2} - \frac{1 - r_i^{*2}}{\ln r_i^*}\ln r^*\right], \tag{8}$$

in which

$$V_m = \frac{Q}{A} = \frac{r_o^2}{8\mu}\frac{\partial p}{\partial Z}\left[1 + r_i^{*2} + \frac{1 - r_i^{*2}}{\ln r_i^*}\right] \tag{9}$$

is the mean velocity and $N_{Pe} = 2Q/[\pi D(r_o + r_i)]$ is the Peclet number. Thus the differential equation is

$$[1 + f(C)]\frac{V_Z^*}{(1 - r_i^{*2})}\frac{\partial C}{\partial Z^*} = \frac{\partial^2 C}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial C}{\partial r^*} + \left[\frac{1}{N_{Pe}(1 + r_i^*)}\right]^2\frac{\partial^2 C}{\partial Z^{*2}} \tag{10}$$

The last term may be neglected since the coefficient, $[N_{Pe}(1 + r_i^*)]^{-2}$, is a very small quantity and $\partial^2 C/\partial Z^{*2}$ is much smaller than its radial counterpart, $\partial^2 C/\partial r^{*2}$, in this geometry. Hence the equation solved in this investigation is

$$[1 + f(C)]\frac{1 - r^{*2} - \dfrac{1 - r_i^{*2}}{\ln r_i^*}\ln r^*}{1 - r_i^{*4} + \dfrac{(1 - r_i^{*2})^2}{\ln r_i^*}}\frac{\partial C}{\partial Z^*} = \frac{\partial^2 C}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial C}{\partial r^*} \tag{11}$$

Correspondingly, the boundary conditions are

$$C(r^*, Z^*) = C_e; \qquad Z^* \leq 0 \tag{12}$$

$$C(r_i^*, Z^*) = C_i; \qquad Z^* > 0 \tag{13}$$

$$C(1, Z^*) = C_o; \qquad Z^* > 0 \tag{14}$$

Equation (11) is a nonlinear, second-order, partial differential equation of the parabolic type. Due to the nonlinearity of the equation, solutions were obtained by numerical methods using a CDC 6400 digital computer.

Although the oxygen concentration is computed at many mesh points for each cross section, the results of the analysis are given in terms of the mixing-cup concentration, the average gas concentration in the fluid exiting the exchanger. The mixing-cup average concentration is the quantity of interest in practical applications. It does, being integrative, smooth out errors induced by some of the assumptions in the analysis, for example, the neglect of coupling between oxygenation and carbon dioxide removal (the Bohr-Haldane effect).† Solutions of oxygenation are obtained in terms of average saturations. At each dimensionless length L^* values of oxygen concentration at different radial points are first converted to local saturation values. The local saturations are then averaged over the cross section according to their respective velocities.

The only parameter in Equation (11) is the ratio of radii r_i^*, that is, the average saturation at a certain dimensionless length L^* depends only on r_i^*. Figure 1 shows the solution obtained for various values of r_i^*. The oxygen concentration at the entrance is assumed to be 7.688×10^{-5} M/l, which corresponds to 75% saturation in the oxygen dissociation curve. Wall concentrations are assumed to be 1.03×10^{-3} M/l, which is equivalent to a partial pressure of 715 mm. Hg. Flow rates being equal, the length required to saturate fully the venous blood in an annulus with $r_i^* = 0.90$ is only 1/34 of that required of a circular tube ($r_i^* = 0$). From $r_i^* = 0.90$ to $r_i^* = 0.99$, another tenfold improvement is gained.

If flow rates are kept constant in annuli of two different sizes but the same r_i^*, the saturation increase after a given length L should be the same. In the larger annulus, blood flows slower; therefore the average residence time of a red cell in the annulus is longer. The oxygen, however, takes a longer time to penetrate the gap of the larger annulus. Thus the two effects cancel each other, leaving the saturation rate independent of the overall size of the annulus.

The solution for human blood ($r_i^* = 0.95$) is shown by the short-dash line in Figure 1. Despite the difference between oxygen dissociation curves for human and cattle blood, the solutions based on these two curves are quite close. Because the solution depends on the slope of the dissociation curve and not on its position, this result is not unexpected.

The concentration (and saturation) profiles across the gap are shown in Figure 2. At points near the walls, pseudo-saturation values over 100% are present. Saturation above 100% merely indicates that the plasma oxygen concentration at these points is above that needed for complete hemoglobin saturation. When a blood sample is collected in a mixing cup, the excess oxygen combines with hemoglobin and further increases the saturation level of the

† An elaborate calculation incorporating the complete Bohr-Haldane effect for a straight tube (7) shows that the effect is small on cup-mixed concentrations.

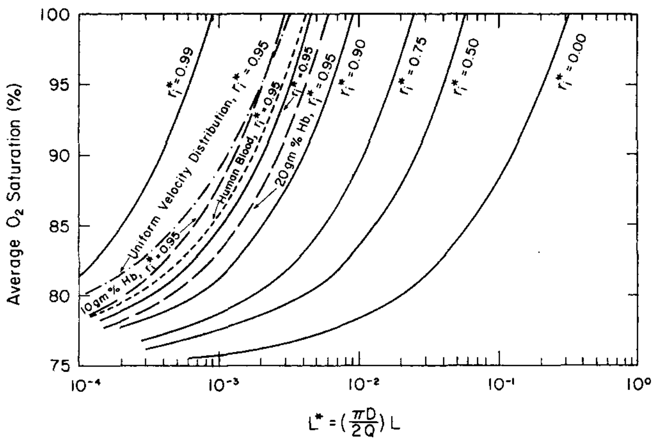

Fig. 1. Average oxygen saturation of cup-mixed samples exiting annular exchangers. The exchange process is assumed to be fluid limited. The flow is axial and laminar. The entrance saturation is 75% and the wall P_{O_2} is 715 mm. Hg. Except as noted, the flow is assumed to be fully developed and the blood has a hemoglobin concentration of 15 g.% and has a bovine oxyhemoglobin dissociation curve.

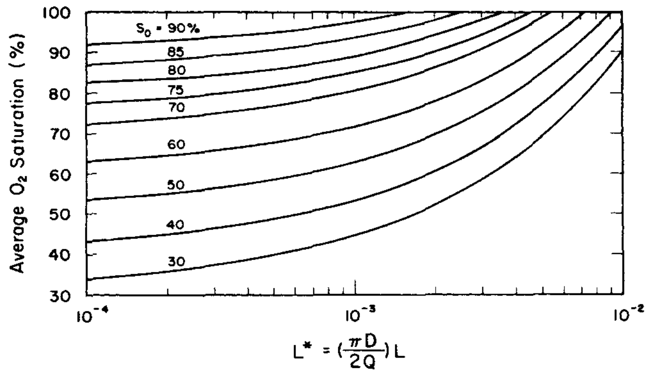

Fig. 3. The dependence of exit sample oxygen saturations on the entrance saturation. The flow is axial, laminar, and fully developed. The exchange process is assumed to be fluid limited, the hemoglobin concentration is 15 g.%, and the wall P_{O_2} is 715 mm. Hg. Radii ratio = 0.95.

sample. If the extra oxygen can cause S% saturation in an equivolumic sample of completely reduced blood, the local saturation is designated as (100 + S)%.

The broken line in Figure 2 indicates the concentration profile at $L^* = 2.5 \times 10^{-3}$ for a flow with a uniform velocity distribution rather than a distribution given by Equation (4). Its proximity to the profile for the fully developed velocity distribution, also at $L^* = 2.5 \times 10^{-3}$, suggests that the solution of Equation (10) is insensitive to the velocity profiles assumed. Indeed, radial molecular diffusion is the predominant feature in this problem and its effect overshadows the effect of axial convection.

As illustrated by the dot-dash line on Figure 1, however, solutions in terms of cup-mixed average saturation are sensitive to velocity profiles. This, of course, is due to the weighting scheme in computing the average saturation; the highly saturated regions near the wall contribute more dominantly to the mixed sample when the velocity profile is uniform than when it is as given by Equation (4).

Normal blood contains about 15 g.% hemoglobin. In cases of anemia or polycyethemia the total hemoglobin content is abnormally low and high, respectively. As illustrated by the long-dash lines on Figure 1 the solution for 10 g.% hemoglobin blood lies above the other two curves because it contains less oxygen sinks and is therefore easier to saturate with oxygen. Additionally, since diffusivity is greater in blood with lower hematocrit values (*8*), the actual difference in the rate of saturation between the 10 g.% hemoglobin blood and the other two samples is even greater than is apparent from the curves. Opposite observations may be made about the polycyethemic blood, that is, 20 g.% hemoglobin.

Solutions obtained from varying the initial saturation are given in Figure 3. The wall concentrations are maintained at 1.03×10^{-3} M/l and $r_i^* = 0.95$ is assumed in these studies.

Finally, the effects of varying the wall concentrations are studied. In Figure 4 solution *A* represents the condition when both cylinders are submerged in normal air. When the oxygen partial pressure is approximately 2 atm. at both

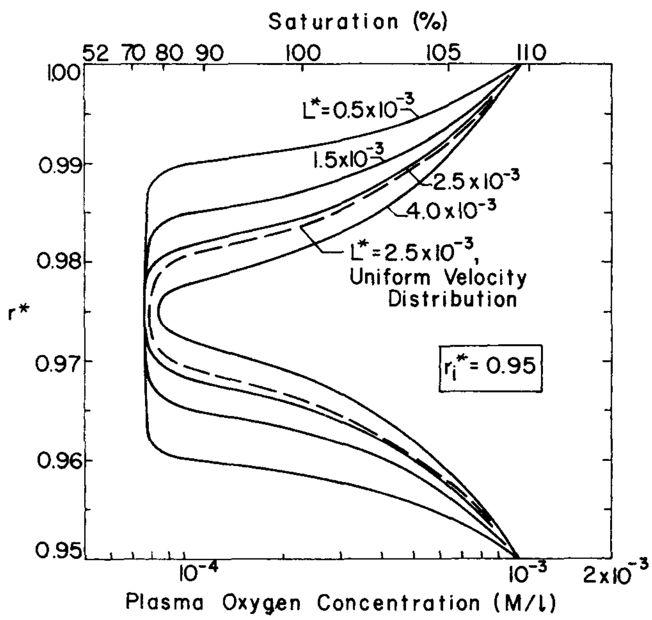

Fig. 2. Concentration profiles in the annular gap. The flow is axial, laminar, and fully developed. The exchange process is assumed to be fluid limited and the wall P_{O_2} is 715 mm. Hg. The radii ratio is 0.95, the hemoglobin concentration is 15 g.%, and the entrance saturation is 75%.

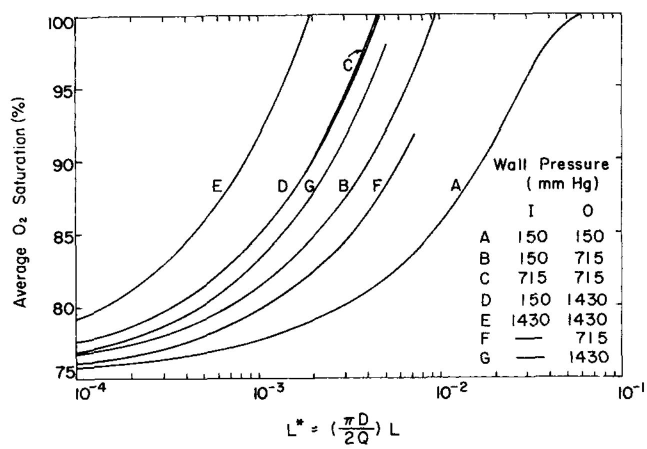

Fig. 4. The dependence of exit sample saturations on the wall P_{O_2} The flow is axial, laminar, and fully developed. The exchange process is assumed to be fluid limited, the entrance saturation is 75%, and hemoglobin concentration is 15 g.%. Radii ratio = 0.95.

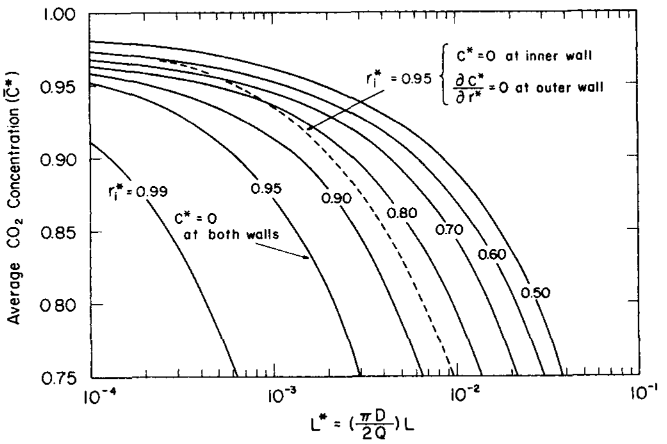

Fig. 5. Average carbon dioxide concentration of cup-mixed samples exiting annular exchangers. The flow is axial, laminar, and fully developed. The exchange process is fluid limited and the entrance concentration is 1.00. The wall concentration is zero at both walls for all curves except the one noted - - -, which has an impenetrable outer wall.

walls, the solution is as shown by line E, which reaches 98% saturation in only 1/25 of the distance as does solution A. Solutions F and G were obtained when the inner wall was made impermeable.

Since $f(C)$ is equal to 9.0 for carbon dioxide removal, Equation (1) can be written as a linear equation

$$10\, V_Z \frac{\partial C}{\partial Z} = D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + \frac{\partial^2 C}{\partial Z^2}\right] \tag{15}$$

Introducing the same dimensionless variables as given in Equations (5) to (8) and

$$C^* = \frac{C - C_w}{C_e - C_w} \tag{16}$$

one may reduce Equation (15) to

$$\frac{10\, V_Z^*}{(1 - r_i^{*2})}\frac{\partial C^*}{\partial Z^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial C^*}{\partial r^*}\right) + \left[\frac{1}{N_{Pe}(1 + r_i^*)}\right]^2 \frac{\partial^2 C^*}{\partial Z^{*2}} \tag{17}$$

By the same argument as used in the oxygenation problem, the last term in Equation (17) can be neglected so that the equation reduces to

$$10\,\frac{1 - r^{*2} - \dfrac{1 - r_i^{*2}}{\ln r_i^*}\ln r^*}{1 - r_i^{*4} + \dfrac{(1 - r_i^{*2})^2}{\ln r_i^*}}\,\frac{\partial C^*}{\partial Z^*} = \frac{\partial^2 C^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial C^*}{\partial r^*} \tag{18}$$

The boundary conditions are

$$C^*(r^*, Z^*) = 1; \qquad Z^* \leq 0 \tag{19}$$

$$C^*(r_i^*, Z^*) = C^*(1, Z^*) = 0 \qquad Z^* > 0 \tag{20}$$

The linear system defined by Equations (18), (19), and (20) can be solved analytically. Viskanta, in studying heat transfer properties in annuli (9), obtained solutions for several values of r_i^* by computing the eigenvalues arising from separation of variables. Since only two of Viskanta's solutions are of interest to this study, additional solutions were obtained numerically. For the values of r_i^* that could be checked, the analytical and numerical results were indistinguishable when plotted. Figure 5 shows the average dimensionless carbon dioxide concentration $\overline{C}^*$ versus L^* for various values of r_i^*. If no carbon dioxide is allowed to diffuse out through the outer wall, the solution is as shown by the broken line on Figure 5.

Since normal arterial P_{CO_2} is 40 mm. Hg and venous P_{CO_2} is 47 mm. Hg, the carbon dioxide in the oxygenator should be removed to such an extent that

$$C^* = [40 - (P_{CO_2})_w]/[47 - (P_{CO_2})_w] \tag{21}$$

If the carbon dioxide pressure is kept zero at the walls, normal arterial condition requires $\overline{C}^*$ to reduce to approximately 0.85.

Figure 6 compares the oxygenation process with the process for carbon dioxide removal. Since the carbon dioxide diffusivity in blood is approximately equal to or slightly greater than the oxygen diffusivity in blood (2), the length needed, with the indicated wall conditions, for oxygenation is the limiting factor in the design of an axial flow oxygenator. In order to prevent over elimination of carbon dioxide, a higher P_{O_2} should be used at the walls or, alternatively, some carbon dioxide should be provided at the walls.

WALL-LIMITED ANALYSIS

In the wall-limited analysis the blood is assumed to have infinite internal lateral mixing. Namely, the oxygen molecules that diffuse into the blood are assumed to be transported laterally to other regions instantaneously, so that a constant oxygen concentration is maintained everywhere in each cross section perpendicular to the flow. As the blood flows through the exchanger and the oxygen content in the blood increases, the pressure gradient across the membrane decreases. Similarly, the carbon dioxide molecules are assumed to be uniformly dispersed at any cross section. The decreased concentration of carbon dioxide in the blood at downstream positions also results in lower gradients downstream.

The analysis of diffusion through the membranes should include the fact that the pressure gradients across the membranes vary with axial position. In the case of oxygen diffusion, the range of this variation is relatively narrow, and, therefore, a constant mean value was used to simplify the analysis. In the case of carbon dioxide diffusion, however, the range of variation is wider, and the total carbon dioxide content is assumed to depend linearly on the P_{CO_2}.

If the axial flow rate is Q cu.cm./min. and the oxygen content in fully saturated blood is C_s M/cu.cm., the per-

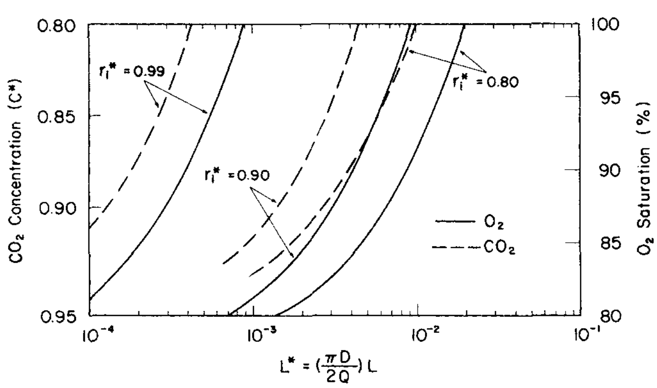

Fig. 6. Comparison of fluid-limited oxygen uptake and carbon dioxide removal for laminar, fully developed axial flow in an annular exchanger. The process is assumed to be fluid limited. The entrance oxygen saturation is 75%, the entrance carbon dioxide concentration is 1.0, the wall P_{O_2} is 715 mm. Hg, and the wall P_{CO_2} is zero. The hemoglobin concentration is 15 g.%.

centage increase in saturation over a length L is, from Fick's law

$$S - S_e = 2\pi\ L(DK)_w \left[\frac{P_{ii} - P_i}{\ln (r_i/r_{ii})} + \frac{P_{oo} - P_o}{\ln (r_{oo}/r_o)} \right] \frac{100}{C_s\, Q}\ \% \tag{22}$$

At the present time, the best material available for use in membrane oxygenators is silicone rubber, and, according to the data furnished by the manufacturers (*10, 11*), $(DK)_w = 1.571 \times 10^{-11}$ M/(min.)(cm.)(mm. Hg). Since the oxygen contained in red cells in fully saturated blood is 8.98×10^{-3} M/l and this constitutes about 98% of the total oxygen in blood, $C_s = 9.1 \times 10^{-6}$ M/cu.cm. The length along the axis is represented by $L = (2Q/\pi D)L^*$, in which D is the oxygen diffusivity in blood and is equal to 6.0×10^{-4} sq.cm./min. (*8*). Combining these relations, the saturation increase after the blood travels a distance L^* is

$$S - S_e = 1.15\ L^* \left[\frac{P_{ii} - P_i}{\ln (r_i/r_{ii})} + \frac{P_{oo} - P_o}{\ln (r_{oo}/r_o)} \right] \% \tag{23}$$

As was mentioned above, P_i and P_o are, strictly speaking, functions of L^*. They may be approximated, however, by a constant mean value without serious error, as the ranges of $P_i(L^*)$ and $P_o(L^*)$ are generally small compared to those of P_{ii} and P_{oo}. If $P_i = P_o = 115$ mm. Hg is assumed and the oxygen pressure outside the oxygenator is kept at 715 mm. Hg, Equation (23) becomes

$$S - S_e = 735\ L^* \left[\frac{1}{\ln (r_i/r_{ii})} + \frac{1}{\ln (r_{oo}/r_o)} \right] \% \tag{24}$$

Thus the saturation increase depends on the size of the cylinders and the thickness of the membranes.

The carbon dioxide outflux in a length dZ is

$$F = -2\pi\ (DK)_w \left[\frac{P_i(Z) - P_{ii}}{\ln (r_i/r_{ii})} + \frac{P_o(Z) - P_{oo}}{\ln (r_{oo}/r_o)} \right] dz\ M/\text{min.} \tag{25}$$

This flux should be equal to the rate of carbon dioxide loss from the blood. Using a linearized dissociation curve and the carbon dioxide solubility coefficient, the rate of carbon dioxide loss may be expressed by $2.51 \times 10^{-7}\ Q\ dP\ M/\text{min.}$, in which Q is the blood flow rate (cu.cm./min.). Since the blood is assumed to have constant carbon dioxide concentration over the cross section, $P_i(Z) = P_o(Z) = P(Z)$. Equating the flux to the loss and integrating yield, with $(DK)_w = 8.33 \times 10^{-11}\ M/(\text{min.})(\text{cm.})(\text{mm. Hg})$ (*11*), $D = 6.0 \times 10^{-4}$ sq.cm./min., and $P_{ii} = P_{oo} = P_w$

$$\ln \frac{P(L^*) - P_w}{P(0) - P_w} = -2.21\ L^* \left[\frac{\ln (r_i/r_{ii}) + \ln (r_{oo}/r_o)}{\ln (r_i/r_{ii}) \ln (r_{oo}/r_o)} \right] \tag{26}$$

If $P(0) = 47$ mm. Hg, $P_w = 0$, the carbon dioxide pressure at L^* is then given by

$$P(L^*) = 47 \exp \left\{ -2.21\ L^* \left[\frac{\ln (r_i/r_{ii}) + \ln (r_{oo}/r_o)}{\ln (r_i/r_{ii}) \ln (r_{oo}/r_o)} \right] \right\} \tag{27}$$

Whether a particular system is fluid limited or wall limited can be determined with the aid of Equations (24) and (27). The desired saturation increase and carbon dioxide pressure drop should be used in the respective equation to solve for L^*; this L^* is then compared with the value obtained in the fluid-limited analysis.

ACKNOWLEDGMENT

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NOTATION

- C = gas concentration
- D = gas diffusivity
- $\partial p/\partial Z$ = axial piezometric gradient
- F = gas flux per unit length of the annulus
- $f(C)$ = oxygen or carbon dioxide sink characteristics in the blood
- K = gas solubility
- L = axial length of exchanger
- N = dimensionless parametric number
- P = gas partial pressure
- Q = flow rate
- r = radial coordinate
- S = percentage of hemoglobin saturated with oxygen
- V = blood velocity
- Z = axial coordinate
- μ = blood viscosity

Subscripts

- CO_2 = carbon dioxide
- e = entrance value
- i = at outer surface of inner cylinder
- ii = at inner surface of inner cylinder
- m = mean value
- O_2 = oxygen
- o = at inner surface of outer cylinder
- oo = at outer surface of outer cylinder
- Pe = Peclet
- s = saturated value
- w = value of variable at wall or value of wall parameter
- Z = axial component

Superscripts

- $*$ = dimensionless variable
- — = average value

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